

# TOM

## *A Multi-touch System for Learning Math*

Daniela Blanke, Georg Schneider

*Dept of Computer Science, University of Applied Sciences Trier, Schneidershof, Trier, Germany*  
*d.blanke@fh-trier.de, g.schneider@fh-trier.de*

**Keywords:** e-learning hardware and software, multi-touch table application, e-learning application for mathematics

**Abstract:** Learning of mathematical concepts is a crucial point for high school students as well as students in science and engineering disciplines. In order to facilitate the learning of these abstract concepts we have developed a system to interactively explore these concepts based on a multi-touch table. The table is used as I/O device to additionally stimulate the haptic sense and to allow simple and intuitive gesture based interaction. Consequently we intend to further improve the learning outcome when using the TOM (Touchable Math) system. In this paper we will describe the concepts and the realisation of TOM, which is a modular framework using Interactive Math Objects, which represent basic mathematical concepts together with exercises related to these objects and a gesture based interaction concept.

## 1 INTRODUCTION

Learning mathematics is a demanding task, especially when the concepts become more abstract and complex. Whereas small children use toys where they move pearls on a bar back and forth for learning counting or simple mathematical operations such as addition and subtraction to literally grasp these ideas, no such support is given for more complex mathematical tasks as middle school students will encounter. Based on findings concerning the process of learning, which we will sketch in the following section our purpose is to transfer these ideas to a learning application for the more advanced mathematical topics.

In the following sections we will start with a motivation of our approach based on results about learning in general. Afterwards we will present in the related work section systems that use a similar approach for learning mathematical concepts. Then we will introduce the concept of our system and describe the realization on a multi-touch table. We conclude the paper with a short summary and an outlook for our future work.

## 2 MOTIVATION

The most natural way to understand our environment is to touch and examine surrounding objects to conceive their meaning. Small children naturally use their hands to discover the world around them and by doing so they make their first learning experiences. Following these principles, in elementary schools the teachers use tangible objects to teach the children in counting and calculating.

The concept of "experiential learning" was described in (Kolb, 1984). Kolb describes the learning as a cycle of concrete experience, reflective observation, abstract conceptualization and active experimentation. This cycle is passed repeatedly and leads the learner to a higher comprehension level. Active experimentation is an important part of this cycle. So the learner can deepen and reinforce the learning contents.

These principles are also applied in applications that use dynamical geometrical contexts to help pupils and students to understand more complex mathematical concepts as well. Many studies show that learning with those applications as applets via internet or as other types of programs provides significant advantage for the learner. The user study from Daher (Daher, 2009) analysing the use of

applets for solving mathematical problems shows that applets are not really necessary but they can help to "feel and see mathematical terms, objects and concepts" (Daher, 2009 p. 389) because they visualize the problems so that these problems can be understood.

The study by (Martinez-Santaolalla, Bienvenida, Tunez, 2005) explores the use of ICT in mathematical education for geometrical problems. They point out that "dynamic geometry software" facilitates specific types of learning activities, for example exploration and visualisation, and can enhance others, such as "proof and proving" (Martinez-Santaolalla, Bienvenida, Tunez, 2005, p. 5).

The use of e-learning platforms in the internet has been explored in the user study of (Albano, Ferrari 2008). The interviewed students valued e-learning as "more interesting" and the learning as "faster and easier".

Regarding the findings of these studies, we can conclude that dynamical maths and the use of specialized software systems are beneficial for pupils and students and help to visualize problems and interact with mathematical terms as described in the principles of Kolb's learning cycle.

In order to stress the fact of interactively exploring mathematical concepts further we came up with the idea to develop a mathematical application using a multi-touch table. Those tables provide an intuitive touch input with multiple fingers instead of the traditional mouse interaction on a PC. So dynamical geometry and the interaction on a multi-touch table can be combined to design a new kind of learning environment for pupils.

### 3 RELATED WORK

Following we will present the most relevant related applications in the field of mathematical e-learning as in regard to our application domain. A part of them belongs to classical PC based programs. We also discuss new applications already using multi-touch tables.

#### 3.1 Mathematical learning programs for PC

The programs "GeoGebra" (Hohenwarter, Fuchs, 2004) and "Cinderella" (Kortenkamp, Richter-Gebert, 2002) aim at a similar direction as our TOM system. GeoGebra is a software for learning

dynamical geometry. Objects can be drawn and altered after drawing. The coordinates of the points and equations are also displayed but only in a subordinate role.

To draw a special object, the kind of the object has to be selected by a button before, i.e. recognition of gestures is not part of the system. Additionally the system can be used to solve mathematical problems, e.g. from a textbook. The system itself does not provide a functionality to integrate exercises for the individual topics, which can be used to help a learner to experience the different facets of a problem and to guide and support the learning process.

The program Cinderella is more comprehensive than GeoGebra. A higher number of objects can be drawn and the system provides more actions to be performed with these objects. The price to pay for this is that a longer training period is necessary to use the entire functionality of the system. To draw the objects and perform actions on these objects, the pupils have to choose the type of the action or object to draw using buttons comparable to GeoGebra.

An additional window must be opened to display equations and the belonging coordinates.

As in GeoGebra the system can be used to solve exercises from a textbook. However integrating exercises into the system is not possible.

Both applications provide a range of functionalities to interactively explore mathematical objects. However the interaction is rather cumbersome. Consequently, an application for dynamical maths on a multi-touch table shall be assessable by simple intuitive gestures. The possibility to integrate exercises shall be envisaged for the system as well.

#### 3.2 Mathematical learning programs on tabletops

First mathematical applications for multi-touch tables have been developed recently.

"Cinderella Multi-touch" is an interactive geometry software on a multi-touch table. This software is an experimental realization of the "Cinderella" system on a tabletop, as described in the previous section. It has been developed by students of the University of Applied Sciences in Schwäbisch Gmünd, Germany (Hochschule für Gestaltung Schwäbisch Gmünd, 2010). The program differs strongly from its PC version.

The target audience for this system are young children. It has been designed to help them understand shape and style of geometrical objects.

For example, the children can learn what a triangle is and that it consists of three points and three connecting lines.

"Smart Table" is a commercial teaching table. (SMART, 2010). According to the informations on the producers web site the table can be used for different educational purposes. A mathematical learning environment for simple mathematical tasks like learning to count or simple geometrical games, like for example Tangram puzzle, are provided. The table is recommended for younger children and so are the applications.

The "Teaching Table" (Khandelwal, Mazalek, 2007) is a table used for pre-kindergarten children. This prototype uses an horizontal fixed LCD screen covered with a sensing grid. This grid gathers the interaction of the pupils with tangible objects on the screen. Some exercises like recognition, assignment and duplication of geometric shapes are implemented as well.

Another approach in the use of mathematical learning programs on tabletops originates from the combination of CAS (Computer Algebra Systems) with a multi-touch interface. The origin were programs like "MathPad2" (LaViola, 2006) and "MathBrush" (Labahn et. al, 2008) that have been built for tablet PCs and were used to write mathematical expressions through handwriting recognition. In these systems the user can write the expression with a pen on the screen instead of typing it in on a keyboard. By writing special symbols the user can trigger calculations or create the graphical representation, which corresponds to the expression. This technique has been recently adapted for tabletops.

"NiCE" (Leitner et. al, 2010) is such a system. It uses an Interpolated Force Sensitive Resistor (IFSR) and can be used together with a desktop or a withboard. Here, pen and multi-touch input are combined. The written expressions can be solved and rearranged by performing multi-touch gestures, like combining two terms together.

Another project is "Hands-On Math" (Zelevnik et. al, 2010). It has been developed for the Microsoft Surface technology. It combines the technique used in "NiCE" with the concept of virtual paper. The expressions can be written on virtual pages and the graphical representations are added to the pages. Calculation gestures like dragging parts of the expression or to split/join them are equivalent to those presented in "NiCE".

The combination of CAS and digital pen and multi-touch interfaces provides an easier use of CAS. However the mathematical expressions can

only be altered but the graphical representation of the expression cannot be manipulated. The graphics can only be scaled but not changed.

Looking at the systems above we can summarize, that current existing mathematical applications on multi-touch tables are either targeted to small children without any relation of graph and expression or they allow the modification of graphs but do not provide sufficient feedback about the corresponding changes of the equation or work. Only the expressions are modified but not the corresponding graph.

The system presented in this paper is based on the idea of creating mathematical objects (like GeoGebra and Cinderella), which can be interactively modified. Different to these systems the objects shall be closely tied to the representation. Objects possess a representation, which is the mathematical formula, that is updated in the same way the objects are manipulated. The intuitive way of interacting with the application shall be similar as in the multi-touch implementations. As it concerns the target audience, we develop the application for maths education in middle schools. For this age, the concepts can be more complex and the learners possess already a certain level of abstraction.

## 4 CONCEPTS OF THE TOM SYSTEM

The basic idea of the TOM system is to develop a mathematical learning software for a multi-touch table that uses the principles of dynamic geometry, which shall help students to learn these concepts easier and to gain a deeper insight in the interrelation of these concepts.

The system should rather be used by pupils to experiment and practise than by a teachers in a classroom situation. So we decided to use a tabletop instead of an interactive whiteboard.

### 4.1 Multi-touch Gestures

In order to incorporate a great amount of interactivity in the system and to guarantee an intuitive usage, all mathematical objects shall be changeable by the user by performing multi-touch gestures. Functions like straight lines, parabolas, vectors etc. shall be drawn by using easy and intuitive gestures. The recognition what to draw shall be done only by the style of the gesture and not

by choosing a special draw option (e.g. click on a "create an ellipse"-button) before.

To remove objects from the screen, also a gesture shall be used. Therefore a simple wipe gesture shall be performed on the object to remove it.

## 4.2 Interactive Math Objects

In addition to the multi-touch gestures it shall be possible, as far as it is mathematical useful, to drag and enlarge the elements or modify them in a specific way. To emphasize the relation between the function drawn and the equation, the equation shall be drawn concise and in the same colour as the graph. If the graph is changed, the equation shall change accordingly.

Therefore we have decided to show both views on the same screen by splitting the screen into a drawing area and an area, in which the equations are rendered. The drawing area is a coordinate system so it is intuitively understandable where to draw the objects because the students already know the concept of a coordinate system from school.

The pupils shall have the option to experiment on their own with the objects, to recognize how the modifications refer to the equations and which part of it is changed when the object is changed in a specific way. For example, if the x radius of an ellipse is changed the corresponding parameter in the equation changes simultaneously. Thus the different parts of such an equation do not have to be learned by heart but can be experienced. In that way the meaning of the equation will be retained much better.

The user shall be able to draw and use objects like straight lines, parabolas, ellipses, vectors and angles. It shall be possible to modify the objects shape and size as far as it is mathematical useful. Besides matrix operations like rotation and mirroring shall be feasible as well. Looking at this topic from the opposite side, it shall also be possible to provide the matrices and to check if the corresponding object behaves as intended.

Furthermore the different objects shall be brought in relation to each other so that things like the slope of a parabola can be understood using the concept of a tangent.

## 4.3 Built-in Exercises

To deepen the learning content, exercises to the different topics shall be provided by the system as

well. These exercises shall be solved by drawing and modifying the mathematical objects. The response shall be communicated in an easy and intuitive way. Therefore the concept of a traffic light is incorporated. The traffic light is red as long as the exercise is not solved correctly. If the answer approaches the correct solution the light will turn to yellow. When the answer is correct the light turns to green and the next exercise can be selected.

In addition persons like tutors or teachers should have the possibility to define own exercises. Therefore a simple authoring tool shall be part of the system where the question text and the test value can be specified.

## 4.4 Tangible objects

A connection to the physical world can be made by using tangible objects. The use of objects equipped with fiducial markers is conceivable. So those objects can be used to identify different users or objects, e.g. a sponge can be used to remove the drawing.

# 5 SYSTEM DESIGN

In the following sections, we will sketch the system design. We will present the hardware used to build the TOM system. Afterwards we will illustrate the realization of the concepts mentioned in the preceding section.

## 5.1 Multi-touch table

The multi-touch table we have used for the development of our system has been developed in our department. It uses the diffused surface illumination (DSI) technology. The touch inputs are gathered by CCV (community core vision) (CCV, 2010) and forwarded to the application by the tangible user interface protocol (TUIO-Protocol) (TUIO.org, 2010).

One of the reasons to build a table on our own is to make use of the DSI technology (Nuigroup, 2010). Likewise it is possible to recognize physical objects that are put on its surface.

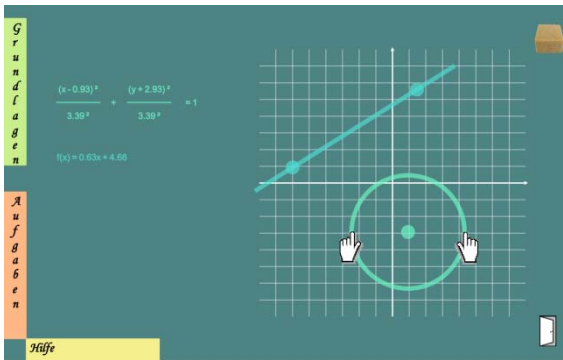


Figure 1: Change x radius (1)

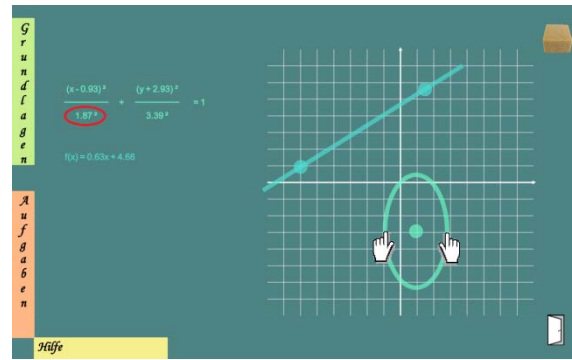


Figure 2: Change x radius (2)

## 5.2 Programming language and framework used

The program has been developed in Java by using the multi-touch framework MT4J (Multi-touch for Java) (MT4j.org, 2010). This framework supports the TUIO protocol and provides basic gesture recognition. Additionally own gestures can be defined.

Besides objects like rectangles, lines, ellipses and polygons that can be used as basic objects for the system graphical representations can be provided implementing an individual view for these objects.

## 5.3 System Design

In general, our design aims at building a modular system in a way that Interactive Math Objects, gestures and exercises can be added individually in order to extend or adapt the system to the special needs of a learning situation.

The user interacts with the system in the context of so called scenes, which are presented to him. In the main scene a coordinate system is displayed. Different listeners are connected to the coordinate system that process the drawing gestures. Objects can be drawn into the coordinate system and in parallel the corresponding equation is displayed in the same colour as the graph. When the drawing is modified the equation changes as well. All objects can only be drawn, moved and modified inside the coordinate system. In the next section the objects and their modification operations are described in more detail.

### 5.3.1 Realization of the Interactive Math Objects

In the following we describe the objects realized yet and the interaction with these objects.

#### 5.3.1.1 Straight Line

A straight line is defined by two points. For this reason we construct a line in our system by touching the surface with two fingers. The straight line is displayed by the two points and a line going through these points.

Afterwards the Interactive Math Object can be modified in different ways. If the user wants to drag the line, he touches the line at an arbitrary position (besides the two distinguished points) and the whole straight line is moved within the coordinate system following the gesture on the table.

Additionally each point can be moved separately and also simultaneously.

Furthermore the straight line can be rotated if the line is fixed at one point and rotated with a second finger. If one point gets out of the coordinate system during the rotation the straight line is shortened so that the point will be kept inside the system.

#### 5.3.1.2 Ellipse

Another Interactive Math Object is the ellipse. The gesture to construct an ellipse is fairly intuitive as well. Like drawing a circle with a compass a circle can be drawn on the table by fixing the centre and moving another finger in a circular motion. In order to facilitate the creation of this object the user must not perform a whole circular movement. A segment of circle is already sufficient, i.e. if the motion describes an angle at about 60 degrees the gesture is recognized and the ellipse with the same value for the x and the y radius is drawn as a point for the centre with a circle line.

The ellipse can be interactively modified as well. If the centre is moved the whole ellipse is moved along the dragging gesture as long as it stays inside the coordinate system. Two fingers are necessary to change the size of the ellipse. If the centre is fixed the circle line can be moved with another finger. Thereby the x and y radius are resized in equal ratio.

Additionally the x and y radius can be changed separately. This is performed when the circle line is pushed or pulled along the x or y axis. Figure 1 and 2 show the changing of the x radius. The hand symbols represent the touching points. In figure 2 the changed equation parameter is marked.

If the x radius should be changed like in the figures above the fingers have to be approximately beside the centre. If the y radius should be changed, the fingers have to be on the circle line approximately above and below the centre.

### 5.3.1.3 Parabola

The third Interactive Math Object that we have realized so far is a parabola. To draw a parabola a three finger gesture is used. To fulfil the condition that three points are necessary to establish a parabola equation. For the correct gesture the apex and two point on the legs of the parabola are defined. These points are then checked if they match the given conditions and are slightly adjusted to match the formula. Afterwards the values for the graph are calculated by using the Gauss algorithm and the graphic view and the corresponding equation are displayed.

Afterwards the apex can be moved up and down and the width and length of the legs can be changed when the points are moved. The legs can only be changed if both points are moved synchronously. The parabola can also be moved through the coordinate system if it is picked by the line. Similar to the other Interactive Math Objects the modification are assigned to the equation.

If the user intends to delete an object he may perform a simple wipe gesture on the object to remove that view and the corresponding equation. To clear the coordinate system and remove all graphs and equations the user can push the sponge-button.

### 5.3.2 Calculation of robust pointing gestures

The precondition for the pointing gestures is that one finger is close to an object. In order to simplify the use of these gestures we use an adjustable threshold,

which allows that a gesture is recognized if the user is "close enough" to the object.

The following code snippet shows the calculation:

```
getFunctionValue(Vector3D p){
    Vector3D pLoc = globalToLocal(p);
    Vector3D cor = new Vector3D(
        (pLoc.getX() - centerPoint.getX()),
        (centerPoint.getY() - pLoc.getY()));

    float v1 = (cor.getX() / radiusX) *
        (cor.getX() / radiusX);
    float v2 = (cor.getY() / radiusY) *
        (cor.getY() / radiusY);
    float value = v1 + v2;
    return value;
}

boolean isOnTheLine(Vector3D p){
    float val = getFunctionValue(p);
    return (0.93 < val && val < 1.07);
}
```

First of all the function value for the drawing object is calculated using the coordinate of one touching point by the function "getFunctionValue". For an ellipse, this value is lower than 1 if the finger is inside the ellipse, greater than 1 if it is outside and exactly 1 if the line got hit. The second function checks if the value is approximately 1.

### 5.3.3 Menus

The systems comes up with three different types of menus: a help menu, an exercise selection menu and a menu that provides mathematical basics. All menus are located at the edges of the screen and look out a little bit like cue cards in a box. This encourages the pupil to pull them out. The menus can only be pulled out up to a specified value likewise it is not possible to push the entire menu under the edge. The basics menu and the exercise selection menu are located at the left edge and can be moved horizontally. The help menu is located at the bottom and can be moved vertically.

The contents of both menus are created dynamically depending on the current objects. So they provide help and mathematical fundamentals to the topics the user is currently interacting with. In addition the help contains general information how to draw and modify objects.

### 5.3.4 Exercises

The program provides exercises on different mathematical topic. These exercises can be chosen by the exercise selection menu. The queries, the

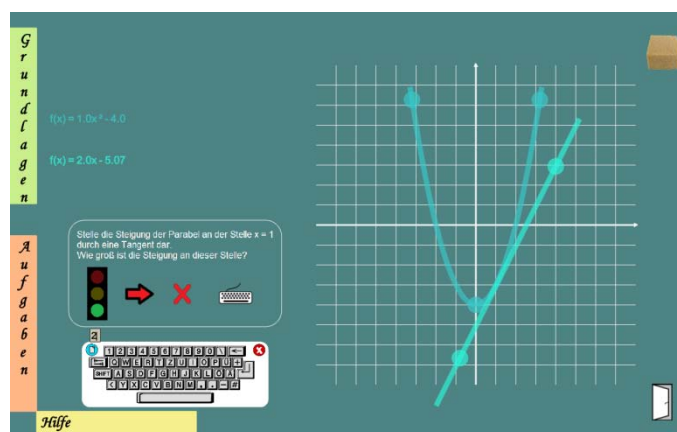


Figure 3: Exercise example

related button and the traffic light symbol are displayed on the left area on the surface. The traffic light shows a red light if the answer is not yet correct, a yellow light if the answer is approximately right and a green light if it is correct. After that the next question can be activated using the arrow button. Some of the exercises require text input. Therefore a soft keyboard can be displayed as input device as well.

Figure 3 shows an example of an exercise. Here the slope of the parabola at the position  $x = 1$  shall be shown with help of a tangent. The tangent is at the correct position and the fact that the answer is right is displayed showing the green traffic light. Afterwards the next exercise can be chosen.

### 5.3.5 Fiducial Marker

Recently we have implemented a feature, which allows to remove objects not only by performing the already mentioned wipe gesture but also by using a normal sponge. This functionality is realized by sticking a printed out fiducial marker under that sponge.

To remove an object the pupil has to put the sponge on the table and wipe over the view in the same way the finger wipe gesture is performed. The use of a real sponge to delete functions is not necessary but it is funny and could support the pupils pleasure while working with the application.

Furthermore an identification of different users by physical objects with fiducial markers will be implemented so that for example already started exercise could be saved and continued without a ordinary user registration.

## 6 CONCLUSION AND FUTURE WORK

Up to now we have implemented the basic functionality of the TOM system. Consequently the system serves as a proof of concept that Multi-touch tables could not only be used for mathematical applications for kindergarten and elementary school but provides advantages for middle school education.

The next steps will be experiments with the users of the targeted user group in order to receive feedback concerning our concepts and implementation.

We want to gather this information in order to direct the future direction of our system, e.g. to figure out if additional functionality is needed on the existing Interactive Math Objects or if we can carry on to explore the interaction with further objects.

## REFERENCES

- Albano, G., Ferrari, P. L. (2008). Integrating technology and research in mathematical education: The case of E-Learning. In Garcia Penalvo (ed.): *Advances in E-learning: Experiences and Methodologies*, 132-148.
- CCV, Community Core Vision, <http://ccv.nuigroup.com>, [21.09.10].
- Daher, W. (2009). Preservice Teachers' Perception of Applets for Solving Mathematical Problems: Need, Difficulties and Functions. *Educational Technology & Society*, 12(4), 383-395
- Hochschule für Gestaltung Schwäbisch Gmünd, Cinderella Multi-touch Wiki, Techert T. et al., [http://ig.hfg-gmuend.de/Members/thomas\\_techert/](http://ig.hfg-gmuend.de/Members/thomas_techert/)

meine-projekte/cinderella-dynamische-geometrie-am-multi-touch, [23.08.2010]

- Hohenwarter, M., Fuchs, K. (2004). Combination of Dynamic Geometry, Algebra and Calculus in the Software System GeoGebra. *Computer Algebra Systems and Dynamic Geometry Systems in Mathematics Teaching Conference 2004*. Pecs, Hungary
- Khandelwal, M. and Mazalek, A. (2007). Teaching table: a tangible mentor for pre-k math education. *Processings of the 1st international conference on Tangible and embedded interaction*, 191–194. ACM Press.
- Kolb, D., (1984). *Experiential learning: Experience as the source of learning and development*. Prentice-Hal.
- Kortenkamp U., Richter-Gebert, J. (2002). Making the move: The next version of Cinderella. *Mathematical Software, Proceedings of the First International Congress of Mathematical Software*, 208–216, World Scientific.
- Labahn, G., Lank, E., MacLean, S., Marzouk, M., Tausky, D., (2008). MathBrush: A System for Doing Math on Pen-Based Devices. *The Eighth IAPR International Workshop on Document Analysis Systems*.
- LaViola, J., (2006). An Initial Evaluation of a Pen-Based Tool for Creating Dynamic Mathematical Illustrations. *Eurographics Workshop on Sketch-Based Interfaces and Modeling*. 157-164.
- Leitner, J., Rendl, C., Perteneder, F., Gokcezade, A., Seifried, T., Haller, M., Zeleznik, R., Bragdon, A., (2010). NiCE Formular Editor. *SIGGRAPH 2010*.
- Martinez-Santaolalla, M. J., Bienvenida, B. F., Tunez, R. S. (2005). ICT in Mathematics Education: geometry problem solving with applets. *Recent Research Developments In Learning Technologies*, La Canada De San Urbano: Department of languages and computation, University of Almeria.
- MT4j.org, MT4J – Multi-touch For Java, <http://mt4j.org>, [21.09.2010].
- Nuigroup, Diffused Surface Illumination - NUI Group Community Wiki, [http://wiki.nuigroup.com/Diffused\\_Surface\\_Illumination](http://wiki.nuigroup.com/Diffused_Surface_Illumination), [21.09.10].
- SMART Technologies, SMART – SMART Table, <http://www.smarttech.de/index.php?page=smart-table-tactile>, [21.09.2010].
- TUIO.org, TUIO, <http://tuio.org/>, [21.09.10].
- Zeleznik, R., Bragdon, A, Adeputra, F., Ko, H. (2010). Hands-on math: a page-based multi-touch and pen desktop for technical work and problem solving. *Proceedings of the 23rd annual ACM symposium on User interface software and technology*. ACM Press.